**Consensus algorithms**

(Hierarchical consensus)

**Example 1** (failure free)

- \( P_1 \)
  - \( \text{propose} \ 1 \)
  - \( \text{broadcast} \ (\text{decided} \ 2, \ a) \)
  - \( \text{doesn't deliver, because} \ r > 1 \)

- \( P_2 \)
  - \( \text{propose} \ 2 \)
  - \( \text{broadcast} \ (\text{decided} \ 2, \ a) \)
  - \( \text{propose} \ 2 \)
  - \( \text{broadcast} \ (\text{decided} \ 3, \ a) \)

- \( P_3 \)
  - \( \text{propose} \ 3 \)
  - \( \text{broadcast} \ (\text{decided} \ 3, \ a) \)

**Example 2**

- \( P_1 \) (\text{decided} \ 1, \ a)

- \( P_2 \)
  - \( \text{propose} \ b \)
  - \( \text{broadcast} \ (\text{decided} \ 2, \ b) \)

- \( P_3 \)
  - \( \text{propose} \ a \)
  - \( \text{propose} \ b \)
  - \( \text{decided} \ 3, \ b \)

**Example 3**

- \( P_1 \) (\text{decided} \ 1, \ a)

- \( P_2 \)
  - \( \text{propose} \ a \)

- \( P_3 \)
  - \( \text{propose} \ a \)
  - \( \text{broadcast} \ (\text{decided} \ 3, \ a) \)

**Proposals:**

- \( a \)
- \( b \)
Correctness

1) Validity
   Each process decides on its proposal or adopted one (proposal of another process)

2) Integrity
   - A node decide only on its round (no more than once)

3) Termination
   - Every correct node terminates at least on its round
     If leader is failed:
     - Perfect failure detector ensures progress
     - B2P broadcast ensures delivery of the leader's correctness

4) Agreement
   No to correct nodes decide differently because they eventually adopt the proposal of a correct leader with minimal ID:
   - No older proposals can overtake the adoption and all future proposals and decisions will be equal to this proposal
   \[ \Rightarrow \text{correctness} \]

5) It is not uniform (example 2: different decisions, because it crashes)